M.Sc.(Physics) Entrance Examination
Utkal University, Bhubaneswar
Symbols have their usual meaning

Section A: Multiple Choice Type

1. Which of the following expresses the Hermite polynomial $H_4(x)$?

   (a) $16\,x^4 - 48\,x^3 + 32\,x^2 + 8\,x$ (b) $16\,x^4 - 48\,x^2 - 36$
   (c) $16\,x^4 + 48\,x^3 - 12\,x$ (d) $16\,x^4 - 48\,x^2 + 12$

2. The value of $\int_{-\infty}^{\infty} x^2\, e^{-\alpha x^2}\, dx$ is

   (a) $\frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}$ (b) $\sqrt{\frac{\alpha^{3/2}}{\pi}}$ (c) $\pi\alpha^3$ (d) $\sqrt{\frac{\pi}{\alpha}}$

3. The integral $\oint_c z^{-2} dz$, where C is the unit circle with centre at the origin and traversed anticlockwise, is

   (a) 0 (b) $-4\pi i$ (c) $2\pi i$ (d) $-i\pi$

4. A dynamical system with two generalized coordinates q_1 and q_2 has Lagrangian $L = \dot{q_1}^2 + \dot{q_2}^2$. If p_1 and p_2 are the corresponding generalized momentum, the correct form of the Hamiltonian for this system is

   (a) $\frac{{p_1}^2}{4} + \frac{{p_2}^2}{4}$ (b) $\frac{{p_1}^2}{2} + \frac{{p_2}^2}{2}$ (c) $\frac{\dot{q_1}^2}{2} + \frac{\dot{q_2}^2}{2}$ (d) $p_1\dot{q}_1 + p_2\dot{q}_2$

5. A particle moves under a central force $F \propto r^n$, where r is the distance of the particle from the force center. Bound state is possible only for

   (a) $-3 < n < -1$ (b) $n = \frac{1}{2}$ (c) $n = 2$ (d) $-1 < n < 0.5$

6. A solid sphere, a solid cylinder and a circular ring all of same diameter, are allowed to roll down a rough inclined plane from same height. Which will reach the bottom first ?

   (a) sphere (b) cylinder
   (c) circular ring (d) all will reach at the same time

7. Which of the following is the orbital angular momentum eigenfunction $Y_l^m(\theta, \phi)$ in a state for which the operators L^2 and L_z have eigenvalues $6\hbar^2$ and $-\hbar$, respectively ?

   (a) $Y_2^1(\theta, \phi)$ (b) $Y_2^{-1}(\theta, \phi)$
   (c) $\frac{1}{\sqrt{2}}[Y_2^1(\theta, \phi) + Y_2^{-1}(\theta, \phi)]$ (d) $Y_3^2(\theta, \phi)$

8. If ψ_n is the n^{th} normalized eigenstate of a one-dimensional simple harmonic oscillator, then the expectation value of the energy operator for the state $\psi(x) = \sqrt{\frac{1}{3}}\,\psi_0(x) + \sqrt{\frac{2}{3}}\,\psi_2(x)$ is

(a) $\frac{1}{6}\,\hbar\omega$ (b) $\frac{1}{2}\,\hbar\omega$ (c) $\frac{5}{3}\,\hbar\omega$ (d) $\frac{11}{6}\,\hbar\omega$

9. The wave function of a particle of mass m moving along the x-axis is given by $\psi(x,t) = A\,e^{-iwt\, -\, \frac{m\,w\,x^2}{\hbar}}$, where A and ω are constants. The potential energy is equal to

(a) $m\omega^2x^2$ (b) $-\,m\omega^2x^2$ (c) $2m\omega^2x^2$ (d) $\frac{m\omega x^2}{\hbar}$

10. $\hat{A}$, $\hat{B}$ and $\hat{C}$ are three Hermitian operators. Which of the following statements cannot be true ?

(a) $[\hat{A},\hat{B}] = \hat{C}$ (b) $[\hat{A},\hat{B}] = i\hat{C}$ (c) $\hat{A}\hat{B} + \hat{B}\hat{A} = \hat{C}$ (d) $\hat{A}^2 + \hat{B}^2 = \hat{C}^2$

11. For a weakly damped oscillating system with resonant angular frequency ω_0 and damping factor η, the quality factor is given by

(a) $\frac{2\pi{\omega_0}^2}{\eta}$ (b) $\frac{\omega_0}{\eta}$ (c) $\frac{\omega_0}{2\eta}$ (d) $\sqrt{\frac{{w_0}^2 - \eta^2}{{w_0}^2 + \eta^2}}$

12. In a plane electromagnetic wave, propagating in vacuum, the electric field is given by $\vec{E}(\vec{r},t) = \hat{x}\,E_0 e^{ik(y+z)-i\omega t}$, where E_0, ω and k are constants. The unit vector along the poynting vector associated with the e.m. wave is

(a) $\frac{\hat{y}+\hat{z}}{\sqrt{2}}$ (b) $\hat{z}$ (c) $\hat{y}$ (d) $\frac{\hat{y}-\hat{z}}{\sqrt{2}}$

13. If d is the distance from the centre of a uniformly charged solid sphere of total charge q, then the electric field inside and outside the sphere are respectively proportional to

(a) d and $\frac{1}{d^2}$ (b) d^2 and $\frac{1}{d^2}$ (c) d^2 and $\frac{1}{d}$ (d) $\frac{1}{d}$ and $\frac{1}{d^2}$

14. A copper rod of length R rotates at angular frequency ω in a uniform magnetic field. The emf developed between two ends of the rod is

(a) constant (b) proportional to R^2
(c) proportional to R (d) proportional to $\frac{1}{R}$

15. In a diatomic gas, the two atoms are joined rigidly to one another. γ (the ratio of C_p and C_v) is

(a) 1.4 (b) 2 (c) 3.5 (d) 2.5

16. The black body spectrum of an object A is such that its radiant intensity ($i.e$ intensity per unit wavelength interval) is maximum at a wavelength of $200\,nm$. Another object B has the maximum radiant intensity at $600\,nm$. The ratio of power emitted per unit area by A to that of B is

(a) $\frac{1}{81}$ (b) $\frac{1}{9}$ (c) 9 (d) 81

17. The Gibbs function for a thermodynamic system does not change in

(a) an adiabatic process (b) an isochoric process
(c) isochoric and isothermal process (d) isothermal and isobaric process

18. The ratio among most probable velocity, mean velocity and root mean square velocity is given by

(a) $1 : 2 : 3$ (b) $1 : \sqrt{2} : \sqrt{3}$
(c) $\sqrt{2} : \sqrt{3} : \sqrt{\frac{8}{\pi}}$ (d) $\sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}$

19. If α and β represent the longitudinal and transversal magnifications respectively of a coaxial optical system placed in an uniform, homogeneous, isotropic medium, then

(a) $\alpha = \frac{1}{\beta}$ (b) $\alpha^2 = \beta$ (c) $\alpha = \frac{1}{\beta^2}$ (d) $\alpha = \beta^2$

20. In a monochromatic beam of light, the electric field is given by $\vec{E}(\vec{r}, t) = [8\,\hat{y}\,sin(kx - \omega t + 65^o) + 8\,\hat{z}\,sin(kx - \omega t - 25^o)]V/m$. The beam of light is

(a) plane polarized (b) elliptically polarized
(c) unpolarized (d) circularly polarized

21. Two coherent monochromatic sources of light have intensities in the ratio $\alpha : 1$. If I_M and I_m denote the maximum and minimum intensities in the interference pattern, then $\frac{I_M + I_m}{I_M - I_m}$ will be equal to

(a) $\frac{1+\alpha}{2\sqrt{\alpha}}$ (b) $\frac{1-\alpha}{\sqrt{2\alpha}}$ (c) $\frac{\alpha}{2\sqrt{\alpha}}$ (d) $\frac{-\alpha}{2\sqrt{\alpha}}$

22. The frequency of rotation of a proton in a cyclotron whose magnetic field is 1 Tesla, (given the mass of proton is 1.67×10^{-27}kg), is

(a) 43 MHz (b) 185 MHz (c) 874 MHz (d) 15 MHz

23. A particle moving with relativistic speed v has an acceleration $\vec{a}$ due to net force $\vec{F}$. If m_0 is the rest mass and $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$, then net force $\vec{F}$ is

(a) $\dfrac{m_0\vec{a}}{\gamma}$ (b) $\dfrac{m_0\vec{a}}{\gamma^{3/2}}$ (c) $\gamma m_0\vec{a}$ (d) $\gamma^3 m_0\vec{a}$

24. Cholesterol deposits reduce the inner diameter of artery of a person by 20%. The percentage increase in pressure exerted by the heart to maintain the same rate of flow of blood is

(a) 40% (b) 160% (c) 86% (d) 144%

25. Given the ionization energy of hydrogen atom is 13.6 eV, the wavelength of the first line of the Lyman series, (given $hc = 1240$ eV nm), is

(a) 64 nm (b) 122 nm (c) 27.2 nm (d) 5000 nm

26. Which of the following system has maximum number of lattice type

(a) cubic (b) tetragonal (c) orthorhombic (d) triclinic

27. A hole in a semiconductor has

(a) zero mass, positive charge
(b) mass same as electron and positive charge
(c) mass opposite to electron and negative charge
(d) mass opposite to electron and positive charge

28. A zener diode of 10 V breakdown voltage is connected in reverse biased condition across a power supply of about 15 V through a series resistor to limit the current to 10 mA. The resistance of the series resistor should be

(a) $2\,k\Omega$ (b) $1.5\,k\Omega$ (c) $1\,k\Omega$ (d) $0.5\,k\Omega$

29. The Boolean expression $(\bar{A}+\bar{B})\bar{C}+\overline{AB}$ simplifies to

(a) $\bar{A}+\bar{B}$ (b) $\bar{A}+\bar{C}$
(c) $\bar{B}+\bar{C}$ (d) $\bar{A}\bar{C}$

30. The process $p \to e^+ + \pi^0$ is forbidden because of violation of

(a) both baryon and lepton number
(b) both charge and lepton number
(c) both baryon number and energy
(d) both lepton number and angular momentum

Section B

1. Calculate $\int_0^{2\pi} \frac{d\theta}{(5 - 3\, sin\, \theta)^2}$.

2. Obtain the Lagrangian for a particle of mass m moving in a plane, under the influence of the potential $V = -\frac{k}{r}$. Find the cyclic coordinates and derive the equations of motion.

3. Φ_1 and Φ_2 are normalized eigenfunctions corresponding to the same eigenvalue. If $\int \Phi_1^* \Phi_2 \, d^3 x = \alpha$, where α is real and positive, find the normalized linear combinations of Φ_1 and Φ_2 that are orthogonal to $\Phi_1 + \Phi_2$.

4. A particle is confined in a one dimensional infinitely deep potential well between $x = 0$ to $x = L$. What would be the probability of finding the particle between $x = 0$ and $x = \frac{L}{4}$. Find the states for which this probability will be maximum.

5. A positively charged particle at rest is released from origin in a uniform electromagnetic field, with $\vec{B}$ pointing in x-direction and $\vec{E}$ in z-direction. Find the trajectory of the particle.

6. A substance X disintegrates into a radioactive substance Y. The initial amounts of X and Y are N_{X0} and N_{Y0}. Calculate the amount of the substance Y after time t.

7. The equation of state of a real gas is given by $P = \dfrac{RT}{V-b}\, e^{-\frac{a}{RTV}}$, where a and b are constants. Find the critical constants of the gas.

8. Pion at rest decays into a muon and a neutrino. Find the energy of the outgoing muon in terms of the two masses m_π and m_μ assuming $m_\nu = 0$.

9. X-ray of wavelength 0.15 nm when incident on the (221) plane of simple cubic crystal, gives a Bragg angle of 30^o. Assuming the order of the fringe to be 1, find the edge-length of the cube.

10. A two stage R-C coupled cascaded amplifier consists of two transistors in common emitter configuration and a fraction of the output of the second stage is fed back to the input of the first stage. The amplification factor of each stage is 5. Draw the circuit of the amplifier and find the feedback factor for which the amplifier will undergo oscillation.